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A Listing and Equating Method for Solving Chemistry/Physics Problems

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ABSTRACT

There are two methods for student exercise in chemistry/physics problem solving; one is a quantity-ratio method (QRM) that uses a ratio of physical quantities of the same kind, and the other is a formalization-and-substitution method (FSM) that substitutes data for physical quantities in formalized equations. This note shows an alternative to the two methods, hereafter called a listing-and-equating method (LEM), for solving chemistry/physics problems. The strategy of LEM is based on the fundamental equation: physical quantity = numerical value × unit. In LEM "listing" lists all the data in a given chemical/physical problem; "equating" equates two physical expressions, each of which is made up of physical quantities and/or numerical values. A two-fold meaning, which often leads chemistry/physics beginners into confusion, in formalized equations can be separated by the listing process. In the equating process of LEM no ambiguity with relation to the equality of QRM occurs. Several examples that are solvable by means of LEM are given.

KEY WORDS
Chemistry/physics problem solving; physical expression; physical equation; physical quantity for equating; listing-and-equating method; SI.

1. Introduction

Let us suppose the following situation in a classroom. **Problem** 1 for student practice is given: What is the mass of salt in 10 mL of a 2 g/L solution?

Some students might use the volume ratio 10 mL/(1 L) in the calculation (2 g) (10 mL/1 L) = 20 mg. This method of calculation is called a quantity-ratio method (QRM) or a factor-label method or dimensional analysis (Malone, 1997). There is a variety of chemical/physical problems that can be solved by use of such a ratio (conversion factor) between physical quantities of the same kind. QRM is applicable to unit conversion, concentration calculation, stoichiometry, and other aspects, in terms of unit ratio, volume ratio, and mole ratio; also refer to Problems 2 and 3.

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The mathematical basis of QRM is a proportion such that m (unknown mass of salt) is to 2 g as 10 mL is to 1 L, namely, m : 2 g = 10 mL : 1 L. This proportional equation leads to m = (2 g) (10 mL/1 L) = 20 mg, which is just the above-mentioned calculation. The foundation of the proportional equality is that the quantity ratios of 20 mg to 2 g and 10 mL to 1 L are the same, but clearly the sum of them has no physical significance. Not all numbers in quantity calculus can be manipulated by the arithmetical rules (addition, subtraction, multiplication, and division) of algebra. The proportion is a mathematical equality that is true or valid for numbers and variables that represent numbers. In other words, the equality relation between quantity ratios of the same kind is ambiguous from the point of view of the rules of physics and/or chemistry.

This note deals with an alternative to QRM, hereafter called a listing-and-equating method (LEM), for solving chemistry/physics problems. The strategy of LEM is based on the fundamental equation (Mills *et al.*, 1993):

$$\text{physical quantity} = \text{numerical value} \times \text{unit}$$

which we will hereafter express as $q = a \times \mathrm{u}$. Here a is not a number in algebra, but a numerical value in chemistry and physics; also refer to the next section and Reference (Morikawa & Nishiyama, 1997). In LEM "listing" is to make a list of data in the form $q = a \times \mathrm{u}$; "equating" is to find equality relationships between physical expressions E and E' in order to work on chemistry/physics problems. Here E is made up of physical quantities and numerical values. Note that E may be or may not be unknown, and that $E = E'$ is reducible to $q = q'$ when E and E' are both physical quantities. LEM adopts only two kinds of equation, $q = a \times \mathrm{u}$ and $E = E'$; no equality relationship of proportion occurs in LEM, and hence the physical and/or chemical meaning of equality in LEM is clear.

If other students in the classroom have learnt about a formalized equation $m = c \times v$, i.e., the mass m of salt is equal to the product of concentration c and volume v, then they could calculate by substituting as follows: m = (2 g/L) × (10 mL) = 20 mg. This method (Malone, 1997) is hereafter called a formalization-and-substitution method (FSM). The formalized equation $m = c \times v$ has a two-fold meaning, the total amount of substance (solution) as in c and a portion of substance as in v; this folding often leads science beginners into confusion (Dierks *et al.*, 1985). LEM separates the two-fold meaning relating to FSM by listing the data of chemistry/physics problems in the form $q = a \times \mathrm{u}$. LEM is a development of FSM.

The outline of two steps, listing and equating, for Problem 1 is as follows.

i. Listing: Write the data in the form $q = a \times \mathrm{u}$. We know c(whole solution) = 2 g/L. Let the unknown mass be m; then the concentration for a 10 mL portion is expressed as c(10 mL portion) = m/(10 mL).
ii. Equating: Search for physical quantities for equating (PQE), and equate two

physical quantities. Clearly the two concentrations are equivalent to each other in the physico-chemical sense; i.e., c(whole solution) = c(10 mL portion), in which PQE is concentration. Then we obtain $m/(10\ \mathrm{mL}) = 2\ \mathrm{g/L}$; this physical equation with the unknown physical variable m represents a physical situation between the whole and a portion of solution.

The remaining work is to get the answer by means of transformation of the physical equation $m/(10\ \mathrm{mL}) = 2\ \mathrm{g/L}$. By setting the unknown physical quantity (amount of substance) to be n, instead of m, one can similarly find the answer to **Problem 2** : What is the molar amount of solute in 10 mL of a 2 mol/L solution? We will discuss LEM in detail.

2. Definition of Physical Expressions in Quantity Calculus

In mathematical equations such as $ax + b = 0$ and $x^2 - 5x + 6 = 0$, each term (and each sum and each difference of two terms) is called an expression (Gellert *et al.*, 1989). Here every letter represents a number. We similarly consider a physical expression E that occurs in chemical/physical problems. The physical expressions in quantity calculus can not be manipulated in the same way as numbers, so a physical expression has to be defined step by step.

Different kinds of quantity (physical quantity, physical unit, physical constant, and numerical value) appear in quantity calculus. Examples of physical quantities are: length, volume, mass, amount of substance, temperature, and concentration. Every physical quantity is a physical expression. A physical unit is a physical quantity that is chosen as the standard quantity (Mills *et al.*, 1993). If a physical quantity q is measured in terms of the unit u, then the result can be expressed as $q/\mathrm{u} = a$ (q divided by u equals a) or as $q = a \times \mathrm{u}$, where a is a numerical value or the measure of q (Guggenheim, 1942). A physical constant is a physical quantity that is measured or defined for worldwide use; so, a physical constant is written as $q = a \times \mathrm{u}$; e.g., the Avogadro constant $N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$. SI (Mills *et al.*, 1993) defines the ice point T_{ice} in Kelvin temperature as $T_{\mathrm{ice}}/\mathrm{K} = 273.15$ or as $T_{\mathrm{ice}} = 273.15\ \mathrm{K}$; i.e., $T_{\mathrm{ice}}/\mathrm{K}$, 273.15, T_{ice}, K, and 273.15 K, are all physical expressions.

Products and quotients of two physical expressions yield new physical expressions (the division by zero is excluded). For example, $(2\ \mathrm{g/L}) \times 10\ \mathrm{mL}$ is a physical expression. It is derived from the division of mL by L such that $\mathrm{mL/L} = 10^{-3}$ (a numerical value) by canceling; both sides are physical expressions.

Not all sums and not all differences of two physical expressions are physical expressions. Let the Greek letter theta be the Celsius temperature; then $\theta/{}^\circ\mathrm{C}$ is a numerical value; hence, $\theta/{}^\circ\mathrm{C} + 273.15$ is a physical expression. Both $\theta/{}^\circ\mathrm{C}$ and v/L are numerical values, but the sum of them is not a physical expression because it has no physical meaning.

3. Making Physical Equations with Unknown Physical Quantities

Two physical expressions E and E' are said to be equivalent only when they have the same dimensions and the same values in a given physico-chemical situation; this equivalence is judged by physical and/or chemical laws. We then link E and E' by the symbol of equality as $E = E'$. It should be noted that E may be or may not be unknown. The physical quantities cited on both sides of $E = E'$ are hereafter referred to as the physical quantities for equating (PQE).

Two expressions 2 g + 3 g (two pieces of metal) and 5 g (a 5 g weight) are equivalent with respect to top pan balances; i.e., 2 g + 3 g = 5 g, in which PQE is mass. Conversion of one unit to another is equivalent; e.g., L = 10 dL = 10^3 mL, in which PQE is volume. A numerical value a in chemical/physical problems is given when a physical quantity q is measured in terms of u; e.g., c(whole solution) = 2 g/L in Problem 1, where PQE is concentration. In Problem 1, we have the equivalence equation $m/(10\ \mathrm{mL}) = 2\ \mathrm{g/L}$ whose PQE is concentration. In SI, thermodynamic temperature T divided by K is equivalent to $\theta/℃ + 273.15$; i.e., $T/\mathrm{K} = \theta/℃ + 273.15$ (Mills *et al.*, 1993).

We thus derive a set of physical equations $E = E'$ with known and unknown physical quantities from a given chemistry/physics problem by means of PQE. In order to solve the problem we must make further equations and use transformation. Several examples employing such transformation are described in the next section.

4. Solving Chemistry/Physics Problems by LEM

Chemical/physical problems in textbooks are usually stated, as shown in Problems 1 and 2, in a native language. Hence, the first task for students is to translate the text into the formalized language of physics or chemistry. LEM translates the data into a list of forms $q = a \times$ u with known and/or unknown physical quantities. We name such a process "listing". The next process is to find equality relationships from the list, and to equate two physical quantities (expressions) as in the forms $q = q'$ and $E = E'$; we call it "equating".

Problem 3 (stoichiometry). What is the amount of NH_3 that can be produced by letting 9 mol of H_2 react with N_2 (Michels, 1993)? LEM proceeds as follows.

i. List the data in the form $q = a \times$ u. We know $n(H_2) = 9$ mol.
ii. Equate two physical quantities in the forms $q = q'$ and $E = E'$. PQE in stoichiometric calculations is the amount of substance. The amount of H is three times the amount of NH_3; $n(\mathrm{H}) = 3n(NH_3)$. The amount of H is twice the amount of H_2; $n(\mathrm{H}) = 2n(H_2)$. It is easy to derive from the three equations that $3n(NH_3) = n(\mathrm{H}) = 2n(H_2) = 2(9\ \mathrm{mol}) = 18$ mol; hence, $n(NH_3) = 6$ mol.

iii. Check the context of the text: Clearly $n(NH_3) = 6$ mol is the answer to the problem.

In the following we cite the steps of QRM (Michels, 1993) to compare LEM with QRM. As shown in Problem 1, QRM is equivalent to solving the proportion, n(unknown): 9 mol of H_2 = 2 mol of NH_3 : 3 mol of H_2.

- Step 1. Write the balanced chemical equation $3H_2 + N_2 = 2NH_3$.
- Step 2. 3 moles of H_2 yield 2 moles of NH_3, so that the mole ratio is written as (2 moles NH_3)/(3 moles H_2).
- Step 3. Multiply the given value 9 moles of H_2 by the mole ratio. Then, (9 moles H_2) × (2 moles NH_3)/(3 moles H_2), gives 6 moles NH_3 by canceling.

Problem 4 (unit conversion). Convert 15 mL to L.

i. Listing: $v = 15$ mL.
ii. Equating: L = 10^3 mL. Then, side-by-side division of the two equations gives v/L = 15 mL/(10^3 mL) = 0.015; then, $v = 0.015$ L; hence, $v = 15$ mL = 0.015 L. Another equating: mL = 10^{-3} L. Then, by substituting one gets v = 15 mL = 15(10^{-3} L) = 0.015 L.

Problem 5 (temperature conversion). Convert 25 degrees Celsius to Kelvin (Atkins, 1990).

i. Listing: $\theta/℃ = 25$.
ii. Equating: $T/K = \theta/℃ + 273.15$. Then, $T/K = 25 + 273.15 = 298$, which leads to $T = 298$ K.

Problem 6 (mole to mass calculation). What is the mass of 2.0 mol of H_2O?

i. Listing: $n = 2.0$ mol. Let m be the unknown mass of H_2O. M(whole substance H_2O) = 18 g/mol, and M(2.0 mol amount of H_2O) = $m/n = m/(2.0 \text{ mol})$.
ii. Equating: PQE is the molar mass; M(2.0 mol amount of H_2O) = M(whole substance H_2O). Then, we get the physical equation $m/(2.0 \text{ mol}) = 18$ g/mol; hence, m = (2.0 mol) (18 g/mol) = 36 g.

Problem 7 (relative atomic mass to amount calculation). The relative atomic mass of Al is 27.0. What is the molar amount of 54.0 g of Al?

i. Listing: M(Al) = 27.0 g/mol, m(Al) = 54.0 g. Let n be the unknown amount of Al. M(amount of Al) = m/n = (54.0 g)/n.

ii. Equating: PQE is the molar mass; M(Al) = M(amount of Al); 27.0 g/mol = (54.0 g)/n; hence, n = (54.0 g)/(27.0 g/mol) = 2.00 mol.

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Morikawa, T., and Nishiyama, Y. (1997) Quantity Calculus in Science Education, *Bulletin of the Joetsu University of Education* 17(1), pp. 365 - 375. In the senior high schools of Japan students (and science textbooks) making use of the proportion (the mathematical basis of QRM in Introduction), m : 2 g = 10 mL : 1 L, are in the minority. Almost all of the students in Japan would get the answer to Problem 1 as follows. Let x be the unknown number; consider given conditions that the solution contains 2 g of salt in 1000 mL, and that 10 mL is in a ratio of 10 to 1000; hence, one can construct a mathematical proportion, $x : 2 = 10 : 1000$; then, to solve it leads to $x = 2(10/1000) = 0.02$; at the last stage the unit "gram" is added as 0.02 [g]. This method used by Japanese students has a property such that the calculation is done on numbers (numerical values) only, and not on physical quantities (each of which equals the product of a numerical value and a unit).